

Title: Alliance Trust Dynamics under Uncertainty: Overcoming the Burden-Sharing Trap

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Abstract:

We develop a continuous-time stochastic differential game within a Linear-Quadratic (LQ) framework to analyze collective deterrence. A dominant Hegemon uses strategic conditionality to link its military commitment to the domestic defence investments of n secondary allies. The alliance's institutional capital is formalized as a dynamic state variable: the trust stock. For analytical tractability, we employ a first-order asymptotic perturbation expansion to derive an explicit, closed-form Markov Perfect Nash Equilibrium. This allows us to establish the exact boundary conditions under which hegemonic conditionality transforms defence spending into strategic complements, preventing exploitation by secondary allies. Furthermore, continuous geopolitical volatility imposes a permanent volatility tax that lowers the expected lifetime welfare of all members and increases the probability of institutional collapse. Although the optimal defence spending rules are not affected by uncertainty, conditionality restructures trust dynamics into a non-linear stochastic process. This serves as an existential failsafe necessary for the alliance's survival under volatility. Counterintuitively, our model reverses the classic 'exploitation of the great by the small': larger alliances derive greater benefit from hegemonic conditionality, driving an accelerated recovery of institutional capital after exogenous shocks. Finally, mapping the alliance's participation constraints establishes that excessive expansion incentivizes hegemonic abandonment, while overly punitive conditionality drives secondary allies to autarky.